Problem 4

For the following series, write formulas for the sequences a_n, S_n, and R_n, and find the limits of the sequences as $n \to \infty$ (if the limits exist).

$$\sum_1^\infty e^{-n \ln 3} \qquad \textit{Hint: } \text{What is } e^{-\ln 3}?$$

Solution

$$a_n = e^{-n \ln 3} = e^{\ln 3^{-n}} = 3^{-n} = \frac{1}{3^n}$$

$$S_n = \sum_{i=1}^{n} \frac{1}{3^i} = \sum_{i=1}^{n} \left(\frac{1}{3}\right)^i = -1 + \sum_{i=0}^{n} \left(\frac{1}{3}\right)^i = -1 + \frac{1 - \left(\frac{1}{3}\right)^{n+1}}{1 - \left(\frac{1}{3}\right)} = \frac{1}{2} - \frac{3^{-n}}{2}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{1}{2} - \frac{3^{-n}}{2}\right) = \frac{1}{2}$$

$$R_n = S - S_n = \frac{1}{2} - \left(\frac{1}{2} - \frac{3^{-n}}{2}\right) = \frac{3^{-n}}{2}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{3^n} = 0$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{3^{-n}}{2} = 0$$